

Narratives of the Experiences of Indonesian Pre-Service Teachers in Learning Mathematics and Their Relationship with Their Beliefs in Mathematics Teaching

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Abstract

Pre-service teachers' experiences of learning mathematics in schools is one of the factors that affect their beliefs on mathematics teaching. This study aims to: (1) explore the learning experiences of Indonesian mathematics pre-service teachers prior to their teacher's training in the university, (2) analyze the connection between pre-service teachers' mathematics learning experiences and their belief related to mathematics teaching. The latter includes beliefs about mathematics, learning mathematics and teaching mathematics. Six first year pre-service mathematics teachers studying in Mathematics Education in a private university in Jakarta were interviewed using an in-depth semi-structured interview where they narrated their experiences in learning mathematics and defined their beliefs related to mathematics teaching. This study explains how the past experiences of the pre-service teachers connect with their beliefs related with mathematics teaching. Our findings showed that most participants experience learning mathematics through listening to teachers' explanations about mathematical concepts and through working on mathematical exercises (drilling). This kind of learning relates with the instrumental and platonic view of mathematics. Consistently, participants in this study show that their beliefs related with mathematics teaching is derived from either the instrumental or platonic view of mathematics, while we did not find any that showed belief from a problem-solving view. This means that the participants believed that mathematics is seen of a set of facts, rules, or skills that is given or it sees mathematics or seen as something fixed and static.

Keywords: Pre-Service Teachers, Belief, Mathematics Teaching

Introduction

By knowing pre-service teachers' experiences in learning mathematics and their beliefs related to mathematics teaching, pre-service teacher education programs can be more informed in designing learning experiences that prepare their students to become better teachers. Beliefs are 'psychologically held understandings, premises, or propositions about the world that are thought to be true' (Philipp, 2007). Pre-service teachers' beliefs related to mathematics teaching can be viewed as lenses that affect how they see mathematics teaching, which is one of the bases for them to make decisions related to mathematics teaching. However, pre-service teachers' beliefs are socially constructed and affected by many other factors (Bronfenbrenner, 1997; Collin & Young, 1986; Schutz et al., 2020). One factor affecting pre-service teachers' beliefs related to mathematics teaching is their previous

experiences in learning mathematics (Lo, 2021). Therefore, studies are needed to explore pre-service teachers' experience in mathematics and their beliefs about mathematics teaching. Unfortunately, there is a lack of study about pre-service teachers' experiences in learning mathematics in Indonesia, especially on how those experiences relate to their corresponding belief in mathematics teaching.

One of the roles of pre-service teacher education programs is to support pre-service teachers to examine beliefs critically concerning the vision of good teaching (Feiman-Nemser, 2001). In the context of mathematics teaching, a vision of good teaching refers to a vision where mathematics teaching engages students to develop their mathematical thinking skills, which includes using their reasoning skills to create mathematical ideas. This vision is based on the belief that mathematics deals with ideas, that mathematical objects are created by humans, especially mathematicians, (Hersh, 1986 in Thompson, 1992) , and that mathematics “is always in the making” (Halmos, 1975; Polya, 1963; Steen 1986 in Thompson, 1992). In other words, pre-service teacher education programs should not be based on the belief that mathematics is seen instrumentally or statically. However, to create those kinds of programs pre-service teacher education should start from “where the students are”. So, it raises question “what are pre-service teachers' beliefs like?”, “where do these beliefs come from?”, “how do their past learning experiences affect their beliefs”?

Before learning to teach in pre-service education programs, all pre-service mathematics teachers are students in school. These pre-service teachers were mathematics learners in school. Therefore, their beliefs about mathematics teaching may be based on being an “apprenticeship of observation” (Lortie, 2020). The apprenticeship of observation describes the process where pre-service teachers learn about mathematics teaching, including the beliefs related to mathematics teaching, through observing their mathematics teachers while they were students, learning mathematics in school. This shows that it makes sense to explore pre-service mathematics teachers' experiences in learning mathematics and analyze how those experiences connect with their beliefs related to mathematics teaching.

There are three main teachers' beliefs that relate to the teaching of mathematics, which includes the beliefs of mathematics, beliefs of teaching mathematics, and beliefs of learning mathematics (Beswick, 2012; Ernest, 1989; Van Zoest et al., 1994). Beswick (2012) synthesizes how all those three beliefs relate with one another by combining the work of Ernest (1989) and Van Zoest et al. (1994) as shown in Table 1.

Table 1. Teachers' Beliefs Related with Mathematics Teaching (Beswick, 2012)

No.	Beliefs About Mathematics (Ernest, 1989)	Beliefs About Teaching Mathematics (Van Zoest et al., 1994)	Beliefs About Learning Mathematics (Beswick, 2012)
1.	Instrumental Mathematics is seen as a set of facts, rules, and skills to be used. The mathematical facts are independent of one another.	Content focus with an emphasis on coverage	Skill mastery, passive reception of knowledge
2.	Platonic Mathematics is seen as a field of study that consists of a set of knowledge that relates with one another. However, the knowledge of mathematics is seen as something that is fixed and static. Mathematics is discovered, not created.	Content focus with an emphasis on understanding	Active constructionism on understanding
3.	Problem Solving Mathematics is seen as a field of study that is created by humans (mathematicians). It consists of a set of knowledge that are related to one another. However, the field of mathematics is growing and mathematics is dynamic.	Learner focused	Autonomous exploration of own interests

This research aims to explore the learning experiences of six Indonesian pre-service mathematics teachers and their beliefs related to mathematics teaching. The research questions for this research are as follows:

1. What are the significant Indonesian mathematics pre-service teachers' experiences in learning mathematics at school?
2. How do the mathematics pre-service teachers' experiences connect with their beliefs related to mathematics teaching?

Method

This research uses a qualitative approach. A qualitative approach is usually used to understand the concepts, opinions or experiences of the participants being researched (Creswell, 2015). Qualitative research is based on the view that social reality is subjective and diverse. The social reality faced by one participant might differ from the other. Consequently, this research focuses on depicting these various social realities from the participants' perspective. This research does not aim to generalize but aims to examine very specific cases (particulars) and explain the specific contexts found in the research, as well as to make meaning of the given data.

This research involves a study of the experiences of prospective teachers, whereby experience is something that is qualitative in nature. Data analysis in this research was therefore carried out interpretively, namely through qualitative analysis. This research also aims to analyze the relationship between prospective teachers' learning experiences and beliefs. This analysis will be carried out qualitatively, which means that the researcher will carry out the process of interpreting various data collected regarding prospective teachers' experiences in learning mathematics and analyzing their relationship with Belief in mathematics, learning mathematics, and teaching mathematics.

More specifically, this research uses narrative research. Narrative research is a type of research that studies human life experiences. Narrative researchers collect these experiences and write narratives related to these experiences (Gudmundsdottir, 2001). In this research, the experiences of prospective teachers will be collected through qualitative in-depth interviews. Interview questions were open-ended as such so that we could see the pre-service teachers' views about certain experiences or phenomena (DeMarrais & Lapan, 2003).

The interview process was carried out using a semi-structured approach. Semi-structured interviews are a method for collecting qualitative data that combines structured and unstructured questions. We prepared a set of open questions that functioned as a guide to the interview process, but still allowed for flexibility in conducting the interview so that we could extract more data from participants (Brinkmann, 2013).

The selection of participants for this research was carried out purposively. All participants came from the same university. Requirements for becoming a participant in this research are as follows: Is a student of the first level mathematics education study program, have the desire to become a mathematics teacher at school, willing to be interviewed in depth

to explore their mathematics learning experiences before entering college. There were six participants in this study, consisting of women from Jakarta and its surroundings. This research was conducted in approximately five months.

The collected data was reduced and analyzed. Data reduction and analysis refers to the process of reducing and summarizing very large amounts of qualitative data so that it can be processed and understood without reducing its essence and main ideas. This data reduction and analysis process was carried out systematically through several processes (Bryman, 2016), namely:

1. Coding: Sorting and classifying raw qualitative data into small groups in a meaningful way
2. Identifying themes: grouping the codes that have been found so that larger themes or patterns can be seen.
3. Summarizing and synthesizing data: selecting appropriate quotes, examples, and narratives to illustrate emerging themes and patterns

Results and Discussion

The significance of Indonesian mathematics pre-service teachers' experiences in learning mathematics at school

Since each participant went to a different school, each had different experiences in learning mathematics. All participants were female went to the same university and were first-year mathematics pre-service teachers. All six came from schools in Jakarta and around Jakarta (Bogor, Bekasi). There are certain similarities between all six participants. They all mention that their dominant experience of learning mathematics is learning through listening to teachers lecturing about mathematical concepts and working on mathematical problems. Nevertheless, each participant had differences in terms of their significant experiences in learning mathematics.

Participant 1 (P1)

P1 went to a *Madrasah Ibtidayah* (Islamic Primary School), then continued her education to a public middle school and a vocational high school. P1 does not remember much of her elementary school experience. While she was in middle school, she met two mathematics teachers whom she enjoyed learning with. She said that both explained mathematics well. They would start their classroom by explaining a certain mathematical

concept. Then students would be given exercises. In turns, students were called one by one to write their mathematical solutions on the board. P1 says:

“When they explain, I understand, I can easily understand the materials.”

During P1’s high school years, P1 learned mathematics online through discord. The teacher would start teaching the class by explaining the materials using an online board. Then, the teacher would give P1 exercises to work on by providing them in Google Classroom.

Participant 2 (P2)

During her school days, P2 always went to public schools. At home, she has a father who loves mathematics. Although the father did not attend university, he always showed curiosity in learning mathematics and would spend time with P2, teaching her about mathematics. P2 says:

“Every day, my father watches mathematics videos on YouTube. He says he enjoys doing mathematical calculations. I thought, if my father could do mathematics, I should be able to do it.”

P2 says that during primary school, she learned mathematics through play. Then, she went to a middle school that was viewed by the public as one of the best public schools in her area. P2 describes that the students in her school were competitive. P2 says:

“The students in my middle school were very ambitious. I felt like I was the only one who was not able to do mathematics. So I thought, come on, the others can do this, so I must be able to do this. I can do this.”

P2 says that most of her experiences of learning mathematics started with her teachers coming to classes and saying, “Today we will be learning [topic]”. The teachers wrote notes on the board. Then students were given time to make notes. Then, the teachers would explain what they wrote about. After that, students were given exercises to work on, usually taken from textbooks. After a while, students would be called to come forward and show their work on the board.

During high school, in grades 11 and 12, P2 was taught by a teacher that she liked. P2 says:

“The 11th and the 12th-grade teacher was the same person. When I asked the teacher about something, she would explain it in detail. When I did not understand something, she would explain so I could understand.”

The teacher also gave the students opportunities to solve problems. P2 says:

“She would go around and check each student’s work and ask each student whether or not they understood. The teacher allowed us to ask questions and try things out. The teacher would sometimes let us struggle with mathematical problems, by not answering a problem. That made me understand the content.”

Some of P2’s experience of learning mathematics were not present. One of P2’s high school teachers did not promote dialogs in the classroom. When the teacher came to the classroom, the teacher showed a PowerPoint and explained the content. The teacher then gave students a piece of paper containing mathematical problems. Sometimes there were 50 problems at once. Students were given a week to work on the problems. P2 says:

“The teacher did not explain the content enough, like the explanation was unfinished so it was not sufficient to solve the problems. We were given grades when we finished the 50-number problems. The teacher did not give the students opportunities to ask questions and never walked around the class to check the students’ work. From the forty students in a classroom, there were usually only five students who did the work. All the others would rewrite the work done by those five people.”

Participant 3 (P3)

P3 went to a public primary school and a private middle school. Both are located in Depok. Then, P3 continued to a private school in Bogor. The high school was held in the *pesantren* (Islamic boarding school). P3 said that she liked mathematics since elementary school. In middle school, P3 met a teacher that she liked. P3 says:

“The teacher taught very well. I felt comfortable. Mostly, everything she taught about, I would understand, although sometimes there are things I do not understand. Whenever I asked her a question, she would spend time answering the question and reviewing the content.”

When learning about a new mathematical equation, P3 always wanted to know the reason that explains why the equation is true. P3 says: “Some teachers explained the content quickly and suddenly said, ‘This is the equation’. I was curious, why is the equation like that? What is the reason behind it? One of my high school teachers would explain the content deeply, elaborating on where the equation came from.”

When P3 was in high school, her peers used to ask her about mathematics. When she did not understand a problem, she would go back and forth to the teacher’s office to discuss

the problem. P3's teacher would explain the problem to her, and sometimes let P3 study in the teacher's office. P3 felt that the teacher understood her needs. P3 said that she had some experience in proving certain mathematical equations. Students were given opportunities to explain the equations that were found in the textbook.

Participant 4 (P4)

P4 went to school in Bekasi. From primary school until high school, P4 always went to public schools. P4 liked mathematics because of the calculations. However, learning mathematics was not always easy for P4. Since primary school, P4 has taken tutoring classes to support her in learning mathematics. When P4 was in elementary school, P4 met a teacher who taught well by providing hands-on experiences. P4 says:

"In elementary school, students were given hands-on experiences, for example when teaching two- and three-dimensional shapes. After that, we would be given mathematical exercises."

In middle school, P4 found it difficult to understand mathematics. P4 took tutoring classes to relearn mathematics. In school, P4 felt like the teacher did not have enough time to answer the students' questions. P4 describes how the teachers taught in middle school:

"The teachers would give a pre-test, explain the content, give us exercises, then give us a post-test. Like there was pressure to learn. In high school, the Covid Pandemic happened. The teachers would explain the content, then give the students mathematical exercises."

Participant 5 (P5)

P5 went to a public primary and middle school, then continued to a vocational high school focusing on accounting. P5 liked mathematics since elementary school because P5 was curious about the mathematical equations she learned about. She was challenged to solve mathematical problems, despite their difficulties. This led her to like mathematics. P5 used Google and YouTube to find out about mathematics. In middle school, P5 had a mathematics teacher that could explain mathematics well. P5 says:

The teacher gave P5 extra mathematical lessons after school. When P5 was not in the 7th grade anymore, the teacher still provided time for P5 to discuss mathematics. P5 says: "Even when the teacher was retired, she would still help me whenever I asked about mathematics."

Participant 6 (P6)

P6 went to a private elementary school, a public middle school, and a high school-level non-formal education program called *Kejar Paket C*, all in Jakarta. P6's mother was an elementary school teacher. In the 6th grade, her mother was P6's classroom teacher. P6's mother would teach P6 mathematics which led P6 to like mathematics. In elementary school, most teachers would teach mathematics by giving explanations while writing on the board. Students would copy the writing. Sometimes the teachers would ask students questions and sometimes use games.

When P6 went to middle school, P6 experienced learning mathematics in groups. Each group was given mathematical problems to solve. Students had to work collaboratively in solving the problems. However, learning mathematics was quite challenging for P6. So, after school, P6 took additional mathematics lessons. P6 says:

“Every day, I took mathematical lessons at a cram school. At home, I also took lessons with a private teacher. It helped me prepare for the national examination so I can be accepted to a prestigious high school.”

When P6 was a student in middle school, a mathematics pre-service teacher did clinical experiences in her school while taking data for her final thesis. P6 was a participant in the pre-service teacher's research. P6 was given a set of mathematical problems to solve. Interaction with the pre-service teacher made her interested in mathematics.

P6 continued her education at a public high school but found it difficult to understand mathematics.

“The mathematical content was more difficult. The teacher explained the students made notes, then the students were given exercises. My grades dropped significantly. I once got 0 on a test.”

P6 decided to move to a different program called *Kejar Paket C*. P6 says: “So, I decided to take the *Kejar Paket C*. There, I joined classes on Saturday and Sunday. From Monday until Friday, I would study on my own by reading books and working on mathematical problems.”

The connections between mathematics pre-service teachers' experiences and their beliefs on mathematics teaching

The process of mathematics teaching and learning should be focused on building students' mathematical proficiency (Kilpatrick et al., 2001), which includes conceptual understanding, strategic competence, procedural fluency, adaptive reasoning, and positive disposition. However, from what we have found in this study, the mathematics learning experiences of all six participants have focused more on procedural fluency. Their experiences with learning mathematics show that the process of learning mathematics comes from listening to the teacher's explanations and working on mathematical exercises. A good mathematics teacher is defined as someone who can explain mathematics well.

P1 sees mathematics instrumentally. She says:

"Mathematics is a study that relates to calculation. Someone can be said to understand mathematics when they know how to use equations for calculating."

P1 believes that teaching mathematics is about the transfer of knowledge. Therefore, the role of the teacher is to transfer this knowledge. Students learn mathematics by using the knowledge to solve mathematical exercises.

P2 learns mathematics by listening to the teacher's explanations and by solving problems. One of P2's teachers lets P2 solve problems by herself. This is called "productive struggle", which refers to the process of letting students think by themselves to solve mathematical problems (Warshauer, 2014). P2 believes that mathematics is a study that relates to calculation, which is useful in daily life. However, P2 also believes that mathematics is seen as a tool for problem-solving. P2's belief seems to come from P2's experience of solving problems. P2 says:

"Someone understands mathematics if they can solve new problems, problems they did not face before. If they are given a new problem and they get confused, it means they do not have the understanding yet."

Although P2 talks about problem-solving, it does not seem that P2's belief in mathematics relates to the problem-solving view. It seems that P2's view of mathematics is that of a platonic view, wherein mathematics consists of a set of concepts that relate to one another but the knowledge is fixed. This can be seen by P2's statement describing what a good mathematics teacher is. P2 says:

“A good mathematics teacher must know the mathematical concepts. If the teacher does not know the mathematical concepts needed for teaching, how will the students master the concept? What if there is a high performing student? What if the teacher cannot answer the students’ question? Would the teacher be ashamed?”

P2’s statement shows that a teacher is someone who knows certain mathematical knowledge and must transfer this knowledge to the students. Therefore, this is in line with the platonic view of mathematics. Students learn mathematics through receiving knowledge from the teacher. Mathematical ideas are not created by the teacher and students together through the process of mathematical discourse.

P3 did have experiences in learning mathematics through listening to teachers’ explanations. However, one of P3’s teachers emphasized explaining the reasoning behind a mathematical statement. Additionally, one of P3’s teachers would explain questions about where an equation comes from and why it is true. P3 says that mathematics is about understanding things, which can include shapes and numbers. Many other things are also related to mathematics, such as natural sciences and economics. P3 explains what it means to understand mathematics:

“Someone who understands mathematics would know where a mathematical equation comes from. For example, someone who understands mathematics would not only know that a circle is pi times its radius square. They would also know why it is so.”

Although P3 described that she experienced proving some mathematical equations while P3 was in school, they also mentioned that the explanations of why a mathematical statement, such as an equation, is true, are learned from the teachers. P3 says:

“I had a teacher whose background was not mathematics. However, the teacher was assigned to teach mathematics. I do not understand the teacher. When I asked the teacher about the reason behind an equation, the teacher could not explain. The teacher told me to ask other teachers.”

That statement shows that P3 believes that mathematics is based on a platonic view. The knowledge of mathematics is seen as fixed. Teachers should know the knowledge of mathematics and transfer the knowledge to students. P3 also believes that mathematics is learned through practice and through learning from an expert. P3 says:

“When someone wants to learn mathematics, they should practice mathematical exercises. They can read about the related content, and everyday practice solving problems related to

that content. I need someone I can consult about my solutions. That person may argue my ideas until I feel enlightened.”

In school, P4 learned mathematics mostly through the process of listening to what the teachers explained, doing mathematical exercises, and taking tests to assess her understanding. P4 also undertook extra mathematics tutoring classes outside of school. P4’s belief about mathematics is that mathematics is about structures and patterns. P4 says: “Mathematics is a field that studies patterns, structures, and changes. Usually, mathematics helps in understanding and explaining concepts related to numbers and operations.”

P4 believes that a mathematics teacher should be knowledgeable and understand the mathematical content that he or she teaches. P4 says: “A mathematics teacher must understand the mathematical content well. A teacher should know far more than the students, so the teacher can explain mathematics well. Additionally, so the classroom situation would not be boring, the teacher can also encourage students to practice mathematics and use more hands-on experiences”

P4 says that she studied mathematics by working on mathematical problems from textbooks. Whenever P4 did not understand a problem, P4 would discuss the problem with her tutor. P4 seems to believe mathematics from a platonic point of view and teachers should be more knowledgeable than students to explain the content of mathematics. P4 can learn mathematics by working on mathematical problems and getting insights from a knowledgeable expert.

P5 believes that mathematics is a study of calculations and equations. P5 also sees mathematics as elegant and simple. P5 likes mathematics because it focuses on logic. P5 believes that learning mathematics should be started from the foundation. P5 says: “When learning mathematics, one should start from the foundation. From operation of numbers, addition, also multiplication. Many find mathematics difficult because they do not understand multiplication and division.”

P5 believes that a mathematical teacher should be knowledgeable and be able to explain mathematics in detail. To study mathematics, one must realize that learning mathematics take time. Then, they must work on mathematical exercises. P5 believes mathematics through a platonic point of view. P5 sees that mathematics is a set of concepts that start from more foundational concepts such as numbers and number operations. A teacher must be able to explain these concepts in detail and students learn mathematics through practice.

P6's experience of mathematics varies. On the one hand, P6 experienced learning mathematics through games, collaborative work, and self-study. On the other hand, P6 also learned mathematics through listening to teachers' explanations. P6 also experienced joining cram schools. P6 did not like learning mathematics from listening to teachers' explanations alone. P6 believes that mathematics does not need rote memorization. It can be studied by using logic. P6 says:

"Someone who understands mathematics can be seen whenever they solve a mathematical problem. They know the reason behind each step of the solution. "

P6 believes that a mathematics teacher must be able to communicate well with the students. "They must also be skillful in mathematics", she says.

Conclusion and Suggestion

This study shows that pre-service teachers' beliefs related to mathematics teaching are connected with their experiences of learning mathematics in school. Preservice teachers who learn mathematics through procedural-focused methods have more rigid views of mathematics. They see mathematics as hierarchical and abstract (Lowrie & Jorgensen, 2016). Participants in this research had beliefs about mathematics from either an instrumental or platonic view. There were no participants who showed beliefs from a problem-solving view. Consequently, their belief about mathematics teaching and mathematics learning reflects this. A mathematics teacher is seen as someone who has fixed knowledge that needs to be transferred to students through the process of explanation. A good teacher is seen as someone who can explain mathematics concepts well. Students learn mathematics by listening to teachers, reading, or through other learning resources such as YouTube. Students are not seen as someone who can discover mathematics through exploration, dialogue, and discourse. Pre-service teachers with experiences of learning mathematics traditionally, where mathematics is seen as a set of fixed rules to be followed, focus more on the importance of procedural fluency rather than conceptual understanding (Ball, 1990). Therefore, universities preparing pre-service teachers in teaching mathematics should acknowledge the pre-service teachers' experiences and support them in learning mathematics. One of those experiences might be by providing pre-service teachers the opportunity to learn mathematics through inquiry-based and constructivist teaching methods (Philipp et al., 2007).

This study can be complemented by quantitative studies regarding mathematics pre-service teachers' beliefs in higher education, with a greater number of samples. Further

studies can also be carried out, for example through longitudinal studies where the development of pre-service teachers' beliefs is studied every year. The aim is to identify changes in pre-service teachers' beliefs regarding mathematics, mathematics learning, and mathematics teaching.

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